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# A Simplified Method for Predicting Satellite Passes

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## Introduction

In spite of the many applications of rise and set (alert) times, elevation and azimuth of a satellite in a polar orbit, the author has found very little reference to detailed derivations of these equations. The present Note uses a much simplified approach to derive these equations for the very special case of plane circular polar orbit about a spherical earth. This is the case of the satellites of the Navy Navigation Satellite System (NNSS). A discussion of the errors introduced by these assumptions is presented at the end of the paper.

## Derivation of the Basic Equation

The rise time calculations are based on solving the orbit equations for the case of an elevation angle $\alpha$ with respect to the station equal to zero and $d\alpha/dt$ greater than zero.

The elevation must be expressed in terms of known parameters. An NNSS satellite defines its own orbit by describing the plane of the orbit with respect to an $XYZ$ system of coordinates.

In this system the $X$ and $Y$ axes define the plane of the equator, with the $X$ axis pointing toward the vernal equinox. The $Z$ axis is directed north along the earth's axis of rotation

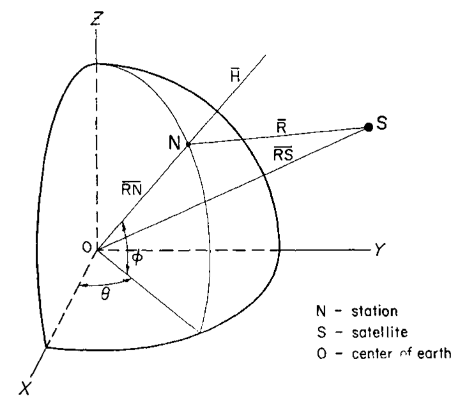


**Fig. 1 Position of station $N$ with respect to the $XYZ$ coordinates.**

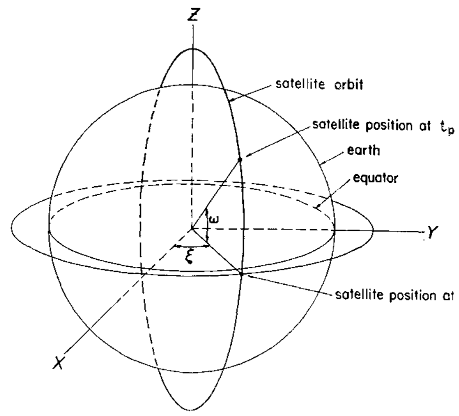


**Fig. 2 The satellite orbit referred to the $XYZ$ coordinates.**

(Fig. 1). This coordinate system does not rotate with the earth.

The station position can also be referenced to the $XYZ$ system by $\theta$, the station sidereal time, and $\phi$, the station latitude. The basic equation will therefore be written, in terms of the $XYZ$ system

$$B = R \sin\alpha = \mathbf{R}\cdot\mathbf{H} \tag{1}$$

where $\alpha$ = elevation angle, measured from the station horizon; $\mathbf{R}$ = slant range vector from the station to the satellite; $R = |\mathbf{R}|$; $\mathbf{H}$ = unit vector at the station pointing toward the station zenith.

The vector $\mathbf{H}$ can be seen from Fig. 1 to have components

$$H_x = \cos\phi \cos\theta,\quad H_y = \cos\phi \sin\theta,\quad H_z = \sin\phi \tag{2}$$

The slant range vector $\mathbf{R}$ can be expressed in terms of the station vector $\mathbf{RN}$ (center of earth to station) and the satellite vector $\mathbf{RS}$ (center of earth to satellite);

$$\mathbf{R} = \mathbf{RS} - \mathbf{RN} \tag{3}$$

$\mathbf{RN}$ has the direction of $\mathbf{H}$ and magnitude $R_e$, the radius of the earth, and is thus related to the $XYZ$ system without further change;

$$\mathbf{RN} = R_e\mathbf{H} \tag{4}$$

To express $\mathbf{RS}$ we should consider the orbit parameters broadcast by the NNSS satellites. These are as follows: $t_p$ = time of perigee, $n$ = mean motion, $\epsilon$ = eccentricity of orbit, $\omega$ = argument of perigee at $t_p$ measured in the orbit plane from the $XY$ plane in the positive $Z$ direction, $\dot{\omega}$ = precession rate of $\omega$, $A$ = semimajor axis of orbit, $\xi$ = right ascension of the ascending node measured from the $X$ axis in a counter-clockwise direction in the $XY$ plane (see Fig. 2), $\dot{\xi}$ = precession rate of $\xi$, $i$ = angle of inclination of satellite orbit, and $\Lambda_G$ = right ascension of Greenwich at $t_p$.

By assuming a circular polar orbit we have set $\epsilon = 0$, $i = 90°$, and $A$ = radius of orbit. The time of perigee $t_p$ and the argument of perigee $\omega$ become meaningless when applied to a circular orbit; so we will define the satellite period $T$ as the time between two successive ascending nodes.

We can now define the orbit plane by an $X_0Y_0$ coordinate system, with $X_0$ in the $XY$ plane making angle $\xi$ with the $X$ axis, and $Y_0$ coincident with the $Z$ axis. $\mathbf{RS}$ can now be written (Fig. 2)

$$\mathbf{RS} = \mathbf{P}X_0 + \mathbf{Q}Y_0 \tag{5}$$

where the transformation vectors $\mathbf{P}$ and $\mathbf{Q}$ relate the orbit plane $X_0Y_0$ to the $XYZ$ coordinate system as follows:

$$\begin{aligned} P_x &= \cos\xi,\quad P_y = \sin\xi,\ P_z = 0 \\ Q_x &= 0,\quad Q_y = 0,\ Q_z = 1 \end{aligned} \tag{6}$$

The coordinates $X_0$ and $Y_0$ can be related to known parameters by

$$X_0 = A \cos E,\quad Y_0 = A \sin E \tag{7}$$

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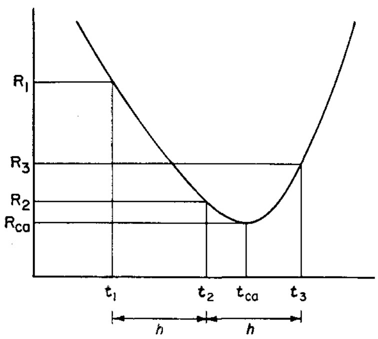


**Fig. 3 Notation for determining the time of closest approach ($t_{ca}$).**

where $E$ = angle between the $X_0$ axis and **RS**.

Combining (6) and (7), **RS** can be rewritten

$$RS_x = A \cos\xi \cos E,\ RS_y = A \sin\xi \cos E,\ RS_z = A \sin E \quad (8)$$

The angle $E$ can also be expressed by

$$E = nt \quad (9)$$

where $n$ = mean motion of satellite, and $t$ = time since the ascending node. The time of the ascending node $t_a$ can be expressed in terms of the broadcast parameters $t_p$, $\omega$, and $n$, as

$$t_a = t_p - \omega/n \quad (10)$$

With all the variables referred to the same coordinate system, we can rewrite the basic equation $B$ for $\alpha = 0$,

$$B = (\mathbf{P}\cdot\mathbf{H})X_0 + (\mathbf{Q}\cdot\mathbf{H})Y_0 - R_e \quad (11)$$

or

$$B = A \cos nt \cos\phi(\cos\xi \cos\theta + \sin\xi \sin\theta) + A \sin nt \sin\phi - R_e \quad (12)$$

Over a short period of time the orbit parameters $\omega$, $\xi$, and $n$ do not vary appreciably. The station latitude $\phi$ also remains constant, but the station sidereal time $\theta$ varies rapidly with time according to the relation

$$\theta_t = \Lambda_{GA} + \lambda + \omega_e(t - t_a) \quad (13)$$

where $\Lambda_{GA}$ = sidereal time of Greenwich at $t_a$, $\lambda$ = station longitude, and $\omega_e$ = constant angular rotation of earth. Expressing $\Lambda_{GA}$ in terms of the broadcast parameters

$$\Lambda_{GA} = \Lambda_G - \omega_e\, \omega/n \quad (14)$$

Substituting (10) and (14) into (13), we get $\theta_t = K + \omega_e t$ where

$$K = \Lambda_G + \lambda + \omega_e t_p \quad (15)$$

Substituting (15) into (12), the basic equation can now be written in the desired form, in $XYZ$ and $t$,

$$B = A \cos nt \cos\phi[\cos\xi \cos(K + \omega_e t) + \sin\xi \sin(K + \omega_e t)] + A \sin nt \sin\phi - R_e \quad (16)$$

Solutions are found to the equation $B = 0$ by stepping $t$ by a small interval until $B$ changes sign. A half-interval search then yields a good solution $t_0$ for $B = 0$.

The stepping interval was chosen as 2 min. This was made as large as possible to reduce scanning time between zero crossings, yet small enough that if the satellite rose for less than 2 min and the zero crossings were missed, the pass lost would not have been of any value for navigation fix computations.

The derivative $dB/dt|_0$ is calculated using

$$dB/dt = A \cos nt[-\omega_e \sin(K + \omega_e t) \cos\phi \cos\xi + \omega_e \cos(K + \omega_e t) \cos\phi \sin\xi + n \sin\phi] - A \sin nt[n \cos(K + \omega_e t) \cos\phi \cos\xi + n \sin(K + \omega_e t) \cos\phi \sin\xi] \quad (17)$$

or

$$dB/dt = X_0[\omega_e(H_x P_y - H_y P_x) + nH_z] - Y_0[n(H_x P_x + H_y P_y)] \quad (18)$$

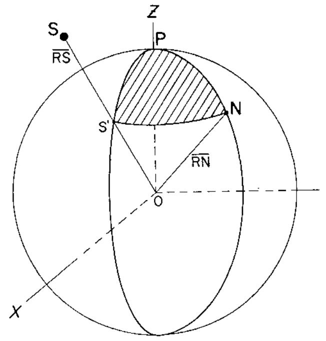


**Fig. 4 Defining the azimuth angle PNS' with respect to the XYZ coordinates.**

### Modifications for Precessing Orbit

Until now we have tacitly assumed that the satellite orbit is stationary in the $XYZ$ system. It does, in fact, precess in both the $\omega$ and $\xi$ directions with known precession constants. In the actual computations the orbit parameters are updated every period.

The mean motion $n$ of the satellite remains constant, independent of the coordinates chosen to represent the orbit. The period, between ascending nodes, however, changes as a function of the precession along the plane of the orbit, or $\dot{\omega}$ so that

$$T' = T + \dot{\omega}T/n \quad (19)$$

The other parameters would then be updated using the new period $T'$,

$$\omega' = \omega + \dot{\omega}T',\ \xi' = \xi + \dot{\xi}T',\ t_a' = t_a + T' \quad (20)$$

### Time of Closest Approach, Elevation and Azimuth

The time of closest approach could be obtained by solving

$$d\mathbf{R}/dt = 0 \quad (21)$$

but this is a cumbersome method. An approximating technique is proposed. Beginning at the rise time, we calculate the slant ranges from (3) at fixed intervals ($h$) until one slant range is larger than the one before. We then assume that the minimum has been passed and fit a polynomial of second order to the three most recently calculated slant ranges (Fig. 3);

$$R = a + bt + ct^2 \quad (22)$$

The time of closest approach $t_{ca}$ is obtained from (20) and (21) as

$$t_{ca} = -b/2c \quad (23)$$

The coefficients $b$ and $c$ are determined by substituting the known data pairs into (21),

$$R_1 = a + bt_1 + ct_1^2,\ R_2 = a + bt_2 + ct_2^2$$
$$R_3 = a + bt_3 + ct_3^3 \quad (24)$$

With perseverance we find that

$$t_{ca} = \frac{R_3(3h - 2t_3) + R_2(-4h + 4t_3) + R_1(h - 2t_3)}{2(R_1 - 2R_2 + R_3)} \quad (25)$$

The elevation $\alpha$ at any time during the satellite pass is simply obtained from (1),

$$\alpha = \text{arc} \sin[(1/R)(\mathbf{RS}\cdot\mathbf{H} - R_e)] \quad (26)$$

The azimuth $\zeta$ of the satellite from station $N$ is represented in Fig. 4 by the spherical angle $PNS'$, where $P$ is the pole nearest $N$, $S'$ is the point on the earth's surface pierced by the vector **RS**.

To solve the spherical angle $PNS'$, we consider the spherical triangle $PNS'$. The sides of this triangle can be represented in angular measure by the plane angles subtended at O, the center of the earth. Thus,

$$\cos PN = \cos PON = \mathbf{ON}\cdot\mathbf{OP}/|\mathbf{ON}||\mathbf{OP}| \quad (27)$$

where **ON** = vector in direction of **H** of magnitude $R_e$, **OP** = vector in $Z$ direction of magnitude $R_e$. Then

$$\cos PN = H_z \tag{28}$$

In a similar fashion

$$\cos NS' = \mathbf{RN}\cdot\mathbf{RS}/|\mathbf{RS}||\mathbf{RN}| = \mathbf{H}\cdot\mathbf{RS}/|\mathbf{RS}| \tag{29}$$

and

$$\cos S'P = \mathbf{RS}\cdot\mathbf{OP}/|\mathbf{RS}||\mathbf{OP}| = RS_z/|\mathbf{RS}| \tag{30}$$

Using spherical trigonometry

$$\cos PNS' = \frac{\cos S'P - \cos PN \cos NS'}{\sin PN \sin NS'} \tag{31}$$

$$\cos\zeta = \frac{RS_z - H_z(\mathbf{RS}\cdot\mathbf{H})}{[\{|\mathbf{RS}|^2 - (\mathbf{RS}\cdot\mathbf{H})^2\}(1 - H_z^2)]^{1/2}} \tag{32}$$

$$\tan\zeta = \frac{RS_xH_y - RS_yH_x}{RS_z(H_x^2 + H_y^2) - H_z(RS_xH_x + RS_yH_y)} \tag{33}$$

### Applications

A FORTRAN program[1] for the Control Data 3100 was written to perform the preceding calculations and output day by day tables of rise time, set time, time of closest approach, and elevation and azimuth at time of closest approach for the NNSS satellites. These tables have been used successfully to predict and identify satellite passes.

The long-range accuracy of prediction is of course based upon the stability of the broadcast precession parameters and the accuracy of the other orbit parameters. Some study was made of the drift over several months. Alert calculations for four days in early March, 1969, were carried out using one set of satellite parameters from March 1969 and one from early June 1968. The rise (and set) times were very consistently 10 min earlier with the March data. The azimuth at $t_{ca}$ was 2° less (average referred to the March data) with a standard deviation of 7°.

The average change in elevation was also 2° with a standard deviation of 7°. The elevation change was greater at greater elevations, thus there were only 5 passes of the 140 predicted in the four days which were not common to both sets of alerts.

It would thus seem that alerts can be predicted for a long time in advance, at least nine months in our experience, and that the rise and set times if corrected by a fixed constant of about −1 min per month, can be used to identify satellite passes.

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# Temperature Predictions during Spacecraft Maneuvers

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### Nomenclature

$A_c$ = projected area of the component whose temperature response is desired
$A_m$ = is the measured area of component 1 in the sketch made with a viewer
$A_1$ = total surface area of component 1
$A_1^{P2}$ = projected area of component 1 to 2
$A_1^{P2'}$ = projected area of component 1, as seen by the illuminated portion of component 2
$A_1^{PS}$ = projected area of component 1 toward the sun
$C_1,C_2$ = $\psi\epsilon_1A_1\sigma$ and $mc_p$, respectively, constants in Eq. (6)
$c_p$ = specific heat of component 1
$F_{1\text{-}2}$ = view factor from 1 to 2
$F_{1\text{-}2'}$ = view factor from component 1 to illuminated portion of 2
$F_{1\text{-}S}$ = "effective" view factor to space. This portion of the energy that leaves 1 does not return (includes energy which bounces off of 2 to space)
$L_c$ = is a component length (see Fig. 3)
$L_m$ = measured length in the sketch corresponding to a known length on a component
$m$ = mass of component 1
$Q_G$ = internal heat generation
$Q_S$ = $\alpha_1A_1^{PS}S$ = direct solar heat input
$Q_R$ = $\epsilon A_1\sigma T_1^4$ = radiated heat from component 1
$Q_{RS}$ = $\rho_2\alpha_1{}^*A_1^{F2'}SF_{1\text{-}2'}$ = reflected solar heat input
$Q_{IR}$ = $\epsilon_1^2\epsilon_1^2A_1^{P2}\sigma F_{1\text{-}2}T_2^4$ = infrared heat input from surroundings
$Q_{\text{stored}}$ = $mc_pdT/d\theta$ = thermal storage of component 1
$S$ = solar constant
$T$ = absolute temperature; $T_i$ = initial value of $T_1$; $T_2$ = effective $T$ of component 2
$\alpha_1,\alpha_1{}^*$ = effective absorptances of 1 to direct from sun and to reflected energy from 2; it is assumed that $\alpha_1{}^* = \alpha_1$
$\epsilon_1$ = effective emissivity of component 1
$\epsilon_1^2,\epsilon_2^1$ = emissivity of component 1 as seen by component 2, and vice versa
$\sigma$ = Stephan-Boltzmann constant
$\psi$ = factor defined by Eq. (3)
$\theta$ = time

#### *Subscripts*

1 = component whose $T(\theta)$ is to be calculated
2 = rest of spacecraft

### Introduction

DURING midcourse or terminal maneuvers, the spacecraft will assume a different position with respect to the sun. Since this position is not known prior to flight, exact thermal predictions cannot be made beforehand. In the past, only worst-case predictions (total eclipse or the worst-case solar heating) were relied upon to determine if a maneuver was acceptable. This is an acceptable answer only if temperature limits are not exceeded. If temperature limits are exceeded with a worst-case analysis, then more accurate predictions are needed. This Note presents a method for quickly but accurately calculating the temperature response of spacecraft components in a nonstandard solar orientation.

### Governing Equation

The equation describing the temperature history of a spacecraft component is

$$Q_S + Q_{RS} + Q_{IR} + Q_G = Q_R + Q_{\text{stored}} \tag{1}$$

Equation (1) can be rewritten so that the direct solar heat input is separate from the other heat input term as follows:

$$\alpha_1A_1^{PS}S - \psi A_1T_1^4 = mc_p(dT/d\theta) \tag{2}$$

Here $\psi$ is a correction factor that incorporates the infrared heat input from the surroundings as well as the reflected solar heat input and internal heat generation. More specifically, $\psi$ is equivalent to

$$\psi = F_{1-S} - (1/\epsilon_1A_1\sigma T_1^4)[\epsilon_1^2\epsilon_2^2A_1^{P2}F_{1-2}\sigma T_2^4 + \rho_2\alpha_1{}^*F_{1-2}'A_1^{P2'}S + Q_G] \tag{3}$$

It will be noted that $\psi$ is a constant if the following realistic conditions are met:

1) The temperature response of the surroundings (2) are about the same as the response of the component of interest (1) and that $T_1$ and $T_2$ are not greatly different.

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